

# Supercurrent in Nodal Superconductors

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In recent years, a number of nodal superconductors have been identified;  $d$ -wave superconductors in high  $T_c$  cuprates, CeCoIn<sub>5</sub>, and  $\kappa$ -(ET)<sub>2</sub>Cu(NCS)<sub>2</sub>, 2D  $f$ -wave superconductor in Sr<sub>2</sub>RuO<sub>4</sub> and hybrid  $s+g$ -wave superconductor in YNi<sub>2</sub>B<sub>2</sub>C. In this work we conduct a theoretical study of nodal superconductors in the presence of supercurrent. For simplicity, we limit ourselves to  $d$ -wave and 2D  $f$ -wave superconductors. We compute the quasiparticle density of states and the temperature dependence of the depairing critical current in nodal superconductors, both of which are accessible experimentally.

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## I. INTRODUCTION

Since the discovery of the organic (TMTSF)<sub>2</sub>PF<sub>6</sub> and heavy fermion CeCu<sub>2</sub>Si<sub>2</sub> superconductors in 1979, the high  $T_c$  cuprate superconductors LaBaCuO<sub>4</sub> in 1986 and Sr<sub>2</sub>RuO<sub>4</sub> and YNi<sub>2</sub>B<sub>2</sub>C in 1994, the new class of nodal superconductors took center stage.<sup>1</sup> However, until recently, except for  $d$ -wave symmetry in high  $T_c$  cuprate superconductors,<sup>2,3</sup> the gap symmetry has not been explored directly for other nodal superconductors.

In the last few years, it has been recognized that the Doppler shift in the quasiparticle spectrum in the vortex state<sup>4,5</sup> provides extremely sensitive means of studying the nodal lines and points in the energy gap  $\Delta(\mathbf{k})$ .<sup>6,7,8,9</sup> More recently the above analysis has been extended to the hybrid  $s+g$ -wave superconductors.<sup>10,11,12</sup> Indeed, through angular dependent magnetothermal conductivity measurements on high quality single crystals at low temperatures ( $T < 1\text{K}$ ), Izawa *et al.* have identified 2D  $f$ -wave superconductivity in Sr<sub>2</sub>RuO<sub>4</sub>,<sup>13</sup>  $d$ -wave superconductivity in CeCoIn<sub>5</sub><sup>14</sup> and  $\kappa$ -(ET)<sub>2</sub>Cu(NCS)<sub>2</sub>,<sup>15</sup> as well as  $s+g$ -wave superconductivity in YNi<sub>2</sub>B<sub>2</sub>C.<sup>16</sup>

However, as seen in Ref. 17, the nature of superconductivity in Sr<sub>2</sub>RuO<sub>4</sub> is still controversial. Clearly both the specific heat data and the magnetic penetration depth data indicate the presence of line nodes in Sr<sub>2</sub>RuO<sub>4</sub>.<sup>8,18,19</sup> Further, both the angular dependent thermal conductivity data<sup>13</sup> and the ultrasonic attenuation data<sup>20</sup> exclude models with vertical line nodes.<sup>21,22</sup> Finally the thermal conductivity data by Izawa *et al.*<sup>13</sup> and Suzuki *et al.*<sup>23</sup> are incompatible with the two gap model by Zhitomirsky and Rice.<sup>24,25</sup> Therefore it is important to understand whether  $f$ -wave superconductivity is realized in Sr<sub>2</sub>RuO<sub>4</sub>. More recent angular dependent magnetospecific heat data for  $T > 0.15\text{K}$  by Deguchi *et al.*<sup>26</sup> do not change this situation, though some of the horizontal nodes may be converted into a set of point minigaps.<sup>27</sup>

In this paper, we concentrate on two superconducting order parameters<sup>1</sup>

$$\Delta(\mathbf{k}) = \Delta \cos(2\phi) \quad d\text{-wave}, \quad (1)$$

$$\hat{\Delta}(\mathbf{k}) = \hat{d} \exp(\pm i\phi) \cos(\chi) \quad f\text{-wave}, \quad (2)$$

where  $\phi$  is the in-plane angle measured from the  $a$ -axis ( $\phi = \tan^{-1}(k_y/k_x)$ ), and  $\chi = ck_z$  with  $c$ , being the  $c$ -axis lattice constant. We study the quasiparticle density of states (DOS) in the presence of superconductivity (SC) and external current, which is accessible through tunneling spectroscopy. We also examine the dependence of critical current in the superconductor as a function of temperature.

Making use of the mean field quasiparticle Green function we can express the quasiparticle density of states in the presence of uniform supercurrent in terms of simple integrals which are evaluated numerically. The superconducting order parameter is also determined within the mean field theory. Then, as in the  $s$ -wave superconductor,<sup>28</sup> the order parameter  $\Delta(T)$  is modified in the presence of supercurrent. We see that the supercurrent first increases linearly with the pair momentum  $\mathbf{q}_s$ , reaches a maximum value, and drops as  $\mathbf{q}_s$  is further increased. We call this maximum value the *depairing critical current* in contrast to the usual critical current associated with depinning of vortices and vortex lattices. The advantage of studying the supercurrent, compared to other transport properties, lies in its sensitivity to the condensate itself.

## II. QUASIPARTICLE DENSITY OF STATES

In the presence of superflow, the quasiparticle Green function in the Nambu-Gor'kov formalism is given by<sup>28,29,30</sup>

$$G^{-1}(i\omega_n, \mathbf{k}) = i\omega_n - \mathbf{v}_F \cdot \mathbf{q}_s + \xi_k \rho_3 + \Delta(\mathbf{k}) \rho_1 \sigma_1, \quad (3)$$

where  $\sigma_i$  and  $\rho_i$  are Pauli matrices acting on the spin and Nambu indices respectively. In the quasi-2D system we are considering<sup>31</sup>

$$\xi_k = \frac{1}{2m}(k_x^2 + k_y^2) - 2t \cos \chi - \mu \quad (4)$$

and  $\mathbf{q}_s$  is the pair momentum.



The quasiparticle density of states in the presence of supercurrent is given by

$$N(E) = -\frac{1}{\pi} \sum_{\mathbf{k}} \text{Im Tr}[G(E, \mathbf{k})], \quad (5)$$

which can be reduced to

$$g(E) \equiv N(E)/N_0 = \text{Re} \left\langle \frac{|E - \mathbf{v}_F \cdot \mathbf{q}_s|}{\sqrt{(E - \mathbf{v}_F \cdot \mathbf{q}_s)^2 - \Delta^2 f^2}} \right\rangle, \quad (6)$$

where  $N_0 = 2m/\pi$ , the function  $f$  is  $\cos(2\phi)$  for the  $d$ -wave and  $\cos(\chi)$  for the  $f$ -wave cases, and the averaging is done over angles  $\langle \cdots \rangle \equiv \frac{1}{(2\pi)^2} \int d\phi \int d\chi (\cdots)$ . For simplicity we shall consider the following four cases.

#### A. $f$ -wave SC with $J \parallel a$ and $d$ -wave SC with $J \parallel c$

The quasiparticle density of states (DOS) is given by

$$g(E, s) = \frac{1}{\pi} \int_0^\pi d\phi g_d(E, s, \phi), \quad (7)$$

where  $s = v_F q_s$  for in-plane current or  $s = 2tq_s \equiv v_F c q_s$  for  $c$ -axis current, and

$$g_d(E, s, \phi) = \frac{2}{\pi} \text{Re} \left\{ K \left( \frac{\Delta}{|E - s \cos \phi|} \right) \right\}. \quad (8)$$

The quasiparticle density of states in the absence of supercurrent for both  $f$ -wave and  $d$ -wave cases is given by  $g_d(E, 0, 0)$ .<sup>30</sup> Here  $K(k)$  is the Complete Elliptic Integral of the First Kind as a function of the modulus<sup>32</sup>  $k$  and  $\text{Re}\{K(k)\} = k^{-1}K(k^{-1})$  for  $k > 1$ .

The quasiparticle DOS is shown in Fig. 1. In the absence of current, say for the  $d$ -wave case, the quasiparticle spectrum has a line of degenerate saddle points along  $\chi$  which give rise to a logarithmic peak in the DOS. At finite current, the Doppler shift  $s \sin \chi$  breaks this degeneracy, leaving only two discrete saddle points. Thus the logarithmic peak at  $|E| = \Delta$  is split into two cusps at  $|E| = \Delta \pm s$ .

#### B. $f$ -wave SC with $J \parallel c$

The integral reduces to

$$g(E, s) = \frac{1}{\pi} \text{Re} \int_{-\pi/2}^{\pi/2} d\chi \frac{|E - s \sin \chi|}{\sqrt{(E - s \sin \chi)^2 - \Delta^2 \cos^2 \chi}}. \quad (9)$$

In the limit  $E \rightarrow 0$ ,  $g(0, s) = \frac{s}{\sqrt{s^2 + \Delta^2}}$ . The quasiparticle DOS is shown in Fig. 2. The Doppler shift  $s \sin \chi$  does not lift the degeneracy of the line of saddle points in the quasiparticle spectrum along  $\phi$ , but does shift them. Thus the logarithmic peak at  $|E| = \Delta$  is shifted to  $|E| = \sqrt{\Delta^2 + s^2}$ . Also, the DOS flattens out for  $|E| < s$  as more low energy states are added near the gap nodes.

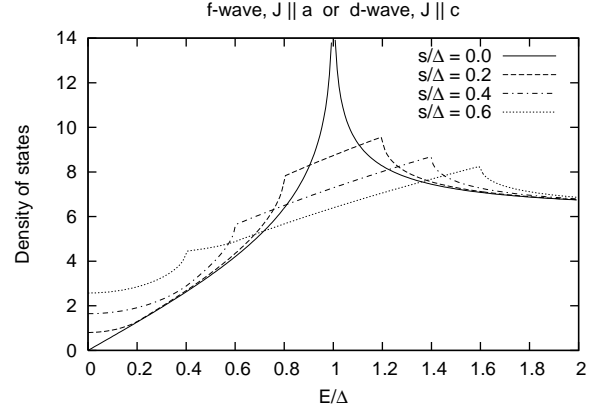


FIG. 1: Quasiparticle DOS  $g(E, s)$  given in Eq. (7).

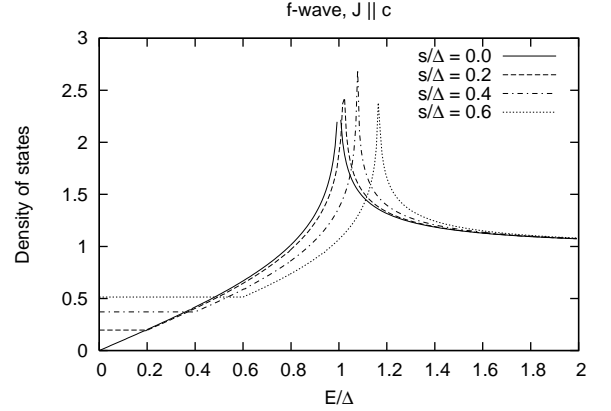


FIG. 2: Quasiparticle DOS  $g(E, s)$  given in Eq. (9).

#### C. $d$ -wave SC with $J \parallel a$

The DOS becomes

$$g(E, s) = \frac{1}{\pi} \text{Re} \int_0^\pi d\phi \frac{|E - s \cos \phi|}{\sqrt{(E - s \cos \phi)^2 - \Delta^2 \cos^2(2\phi)}}. \quad (10)$$

When  $E = 0$ , we find

$$g(0, s) = \frac{s}{\pi \Delta} \text{Re} \left\{ \frac{1}{\sqrt{a}} K \left[ \sqrt{\frac{b}{a}} \right] \right\}, \quad (11)$$

where  $b = s\sqrt{s^2 + 8\Delta^2}/(4\Delta^2)$  and  $a = (1 + b)/2 - s^2/(8\Delta^2)$ .

The quasiparticle DOS is shown in Fig. 3. Just as in the previous section, the logarithmic peak at  $|E| = \Delta$  is shifted to  $|E| = \sqrt{\Delta^2 + s^2/4}$ , while secondary peaks appear at  $|E| = \Delta \pm s$  due to the interference of the gap  $\Delta \cos(2\phi)$  and Doppler shift  $s \cos \phi$  modulations.



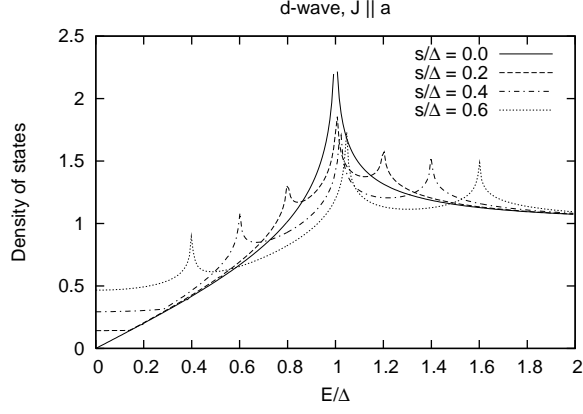


FIG. 3: Quasiparticle DOS  $g(E, s)$  given in Eq. (10).

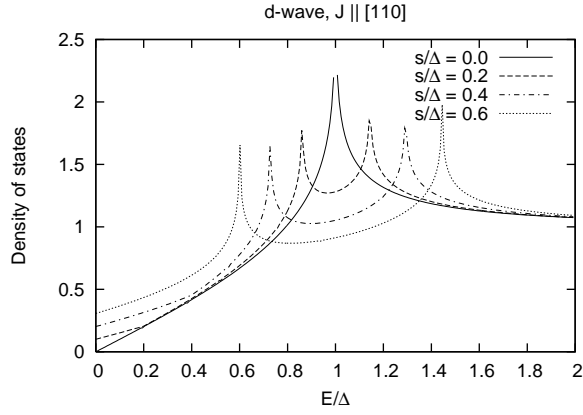


FIG. 4: Quasiparticle DOS  $g(E, s)$  given in Eq. (12).

#### D. *d*-wave SC with $J \parallel [110]$

The DOS is given by

$$g(E, s) = \frac{1}{\pi} \text{Re} \int_0^\pi d\phi \frac{|E - s \cos \phi|}{\sqrt{(E - s \cos \phi)^2 - \Delta^2 \sin^2(2\phi)}}. \quad (12)$$

For  $E = 0$ , we find

$$g(0, s) = \frac{2}{\pi} \text{Re} \left\{ K \left( \frac{2\Delta}{s} \right) \right\}. \quad (13)$$

The quasiparticle DOS is shown in Fig. 4. As in the previous section, logarithmic singularities persist, but the main peak at  $|E| = \Delta$  is split into two smaller ones approximately at  $|E| = \Delta \pm s/\sqrt{2} + \mathcal{O}((s/\Delta)^2)$  for small  $s/\Delta$ .

### III. GAP EQUATION AND CRITICAL CURRENT

Within BCS theory, the gap equation is given by

$$\Delta(\mathbf{k}) = T \sum_{i\omega_n} \sum_{\mathbf{p}} V_{\mathbf{k}\mathbf{p}} \text{Tr}[\rho_1 \sigma_1 G(i\omega_n, \mathbf{p})], \quad (14)$$

where  $\Delta(\mathbf{k}) \equiv \Delta f(\mathbf{k})$  and  $V_{\mathbf{k}\mathbf{p}} \equiv V f(\mathbf{k}) f(\mathbf{p})$ . This equation can be reduced to

$$-\frac{1}{2} \ln \left( \frac{\Delta}{\Delta_0} \right) = \text{Re} \left\langle f^2 \cosh^{-1} \left( \frac{sz}{\Delta f} \right) \right\rangle \quad (15)$$

at  $T = 0$ , where  $z = \cos(\phi - \phi_0)$  or  $z = \sin \chi$  for the in-plane or *c*-axis current respectively, with  $\phi_0$  being the direction of the in-plane current.  $\Delta_0$  is the order parameter at  $T = 0$  and  $\mathbf{q}_s = 0$ . At zero temperature, the *f*-wave case was previously investigated in Ref. 29, while the *s* and *d*-wave cases were considered in Ref. 41.

The dependence of the order parameter on  $s$  is shown in Fig. 5. Unlike in the 3D *s*-wave superconductor, both the order parameter  $\Delta$  and the supercurrent  $j_s$  jump (see Fig. 6) at  $s/\Delta_0 \sim 0.8$ , except for the case of an *f*-wave superconductor with *c*-axis current. The values of  $\Delta$  obtained via the gap equation and  $j_s$  are shown for  $s$  increased from zero. In all cases the jump disappears at a finite temperature and the transition to normal state becomes continuous. These temperatures are  $0.06\Delta_0$  and  $0.25\Delta_0$  for the *d*-wave superconductor with current along the *a* and  $[110]$  axes respectively. The transition is continuous for all  $T > 0$  in the *f*-wave case with in-plane current.

This behavior is strongly reminiscent of the case of Pauli paramagnetism in both the *s*-wave and *d*-wave superconductors.<sup>28,33</sup> These jumps induced by the Pauli term signal the presence of more stable inhomogeneous superconductivity, the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state.<sup>34,35</sup> Very recently, presence of the FFLO state has been reported from thermodynamic,<sup>36</sup> ultrasonic,<sup>37</sup> and temperature dependent upper critical field measurements<sup>38</sup> in *d*-wave superconducting CeCoIn<sub>5</sub>.<sup>14</sup>

Therefore, we speculate the appearance of inhomogeneous superconductivity in the vicinity of  $s/\Delta_0 \sim 0.8$  similar to the FFLO state, but generated by uniform supercurrent, in all the cases we have considered except for an *f*-wave superconductor with *c*-axis current. A more detailed analysis of the free energy and possible FFLO state will be reported in the near future.

The finite temperature gap equation reduces to

$$-\frac{1}{2} \ln \left( \frac{\Delta}{\Delta_0} \right) = \text{Re} \left\langle f^2 \cosh^{-1} \left( \frac{sz}{\Delta} \right) \right\rangle + \left\langle f^2 \int_{\Delta|f|}^\infty dE \frac{h(E + sz) + h(E - sz)}{\sqrt{E^2 - \Delta^2 f^2}} \right\rangle, \quad (16)$$

where  $h(\varepsilon) = n_F(\varepsilon) - n_F(\varepsilon, T = 0)$ , with  $n_F(\varepsilon)$  being the Fermi distribution.



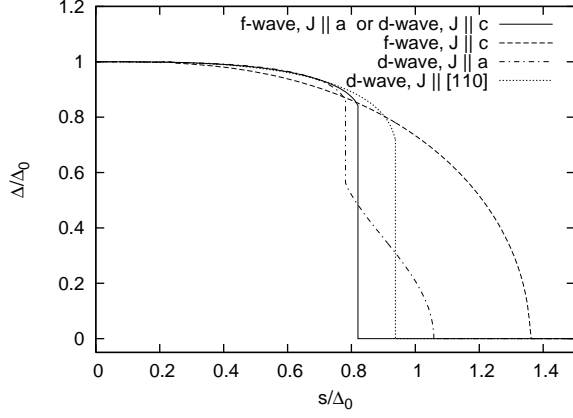


FIG. 5: Zero temperature order parameter as function of  $s$ , as defined by Eq. (15).

Finally, we study the behavior of the supercurrent, which can be evaluated as follows

$$\mathbf{j}_s = \frac{ne\mathbf{q}_s}{m} + T \sum_{i\omega_n} \sum_{\mathbf{k}} e\mathbf{v}_{\mathbf{k}} \text{Tr}[G(i\omega_n, \mathbf{k})], \quad (17)$$

and reduces to

$$j_s = J_0 s \left( 1 - 2 \text{Re} \left\langle z^2 \sqrt{1 - \frac{\Delta^2 f^2}{(sz)^2}} \right\rangle \right) \quad (18)$$

at  $T = 0$ , where  $J_0$  is a constant differing by a factor of  $v_{Fc}/v_F$  between the in-plane and  $c$ -axis cases. Solving Eq. (16) for given  $s$ , and substituting the result into Eq. (18), we can also evaluate  $j_s$  for given  $s$ . The behavior of the supercurrent as a function of  $s$  is shown in Fig. 6. For the  $f$ -wave case, an analytic expression for the zero temperature supercurrent and the depairing current  $j_{\text{max}}$  (at which  $dj_s/ds = 0$ ) have been evaluated in Ref. 29. Also, the current curves for the  $s$  and  $d$ -wave cases were numerically evaluated in Ref. 41. As expected, for large  $s$  the equilibrium value of the current is zero since superconductivity is destroyed. For small  $s$ , the current rises linearly, it then reaches a maximum, and finally decreases to zero in the same way as the order parameter.

The finite temperature expression for the current becomes

$$j_s = J_0 s \left( 1 - 2 \text{Re} \left\langle z^2 \sqrt{1 - \frac{\Delta^2 f^2}{sz^2}} \right\rangle + 2 \left\langle z^2 \int_{\Delta|f|}^{\infty} \frac{(h(E+sz) - h(E-sz)) E dE}{\sqrt{E^2 - \Delta^2 f^2}} \right\rangle \right). \quad (19)$$

The dependence of the depairing current  $j_{\text{max}}$  on temperature is illustrated in Fig. 7. The behavior of the critical current as a function of temperature is qualitatively similar for all of the considered cases except for the

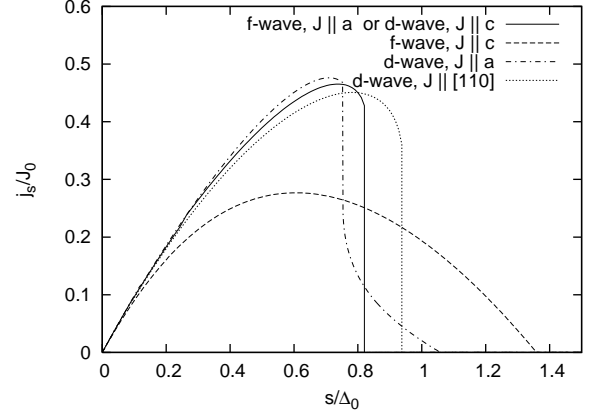


FIG. 6: Zero temperature supercurrent as a function of  $s$ , given in Eq. (18). Note that the  $J_0$  differs by a factor of  $v_{Fc}/v_F$  between the in-plane and  $c$ -axis cases.

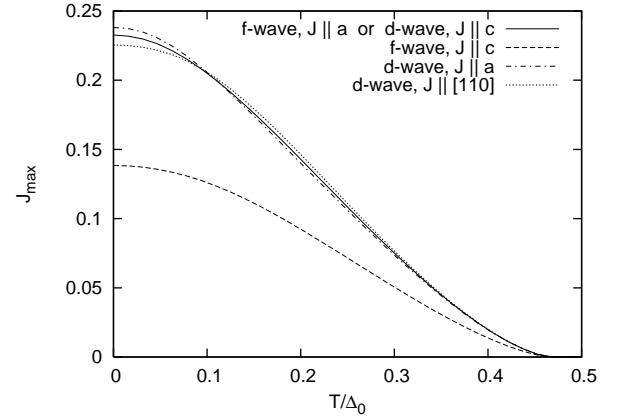


FIG. 7: Depairing critical current as a function of temperature, defined by the maximum value of  $j_s$  (Eq. (19)) for given  $T$ .

$f$ -wave SC with current along the  $c$ -axis. This case is set apart in the same way as when the behavior of the order parameter was considered.

#### IV. CONCLUSION AND SUMMARY

We consider the supercurrent as a simple pair breaking perturbation which adds a Doppler shift to the quasi-particle spectrum. While this study is based on the assumption of uniform supercurrent throughout a sample, the experimental realization of uniform current is far from trivial.<sup>42,43</sup> In bulk crystals, the supercurrent will be localized near the sample surface due to the Meissner effect. Therefore, it is important to have high quality thin films or whiskers of relevant compounds. Very recently, whiskers of Bi2212 have been used to measure the cross junction Josephson tunneling,<sup>39,40</sup> which implies that this experiment, while nontrivial, should be possible in the future.



As long as the condition of uniform current is met, the quasiparticle density of states obtained here will be readily accessible by scanning tunneling microscopy and other tunneling techniques. The temperature dependence of the critical depairing current may also be measured. These measurements will provide useful information on the pairing symmetry of nodal superconductors.

As we have seen, the DOS is very sensitive to the symmetry of the gap and the direction of supercurrent. In principle, the DOS is sensitive to the location and type of critical points in the quasiparticle dispersion. These critical points are easily perturbed or displaced in different ways by adding to the dispersion an appropriate term linear in electron velocity—the Doppler shift. Thus, with the addition of a uniform current, the change in the singularities of the DOS can reveal a lot about the symmetry of the unperturbed quasiparticle dispersion.

Also, in many cases, the spatially homogeneous configuration as considered here becomes unstable for  $s/\Delta_0 \sim 0.8$ . This opens up the possibility of a current induced Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state. This

possibility will be further explored.

In summary, we have evaluated the effect of the presence of finite supercurrent on the quasiparticle DOS of nodal superconductors. The quasiparticle DOS can be probed by tunneling spectroscopy and is sensitive enough to the order parameter and supercurrent direction to help differentiate between superconductors of different gap symmetries. In addition, we showed the dependence of the supercurrent on the external current and the dependence of the critical depairing current on temperature.

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